2. Klausur

1. Aufgabe: $f(x,y) = \sqrt[3]{x} + \sqrt[5]{y} = x^{\frac{1}{3}} + y^{\frac{1}{5}} \qquad x,y > 0$

i) $\frac{\partial f}{\partial x}(x,y) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3}\frac{1}{\sqrt[3]{x^2}} \qquad \frac{\partial f}{\partial y}(x,y) = \frac{1}{5} y^{-\frac{4}{5}} = \frac{1}{5}\frac{1}{\sqrt[5]{y^4}}$

$df(x,y) = \frac{1}{3} x^{-\frac{2}{3}} dx - \frac{1}{5} y^{-\frac{4}{5}} dy$

ii) $\sqrt[3]{1.03} + \sqrt[5]{0.975} = f(1.03; 0.975) \approx f(1,1) + \frac{\partial f}{\partial x}(1,1)\cdot 0.03 + \frac{\partial f}{\partial y}(1,1)\cdot(-0.02$

$= 2 + \frac{1}{3} 0.03 - \frac{1}{5} 0.025 = 2 + 0.01 - 0.005 = 2.005$

nebenbei: exakt ist 2.004851

2. Aufgabe i) $\int_0^{\pi} \sin x \, dx = \left[-\cos x\right]_0^{\pi} = 1 + 1 = 2$

ii) $\int_{-1}^{1} \cosh u \, du = 2\int_0^1 \cosh u \, du = 2\left[\sinh u\right]_0^1 = 2\sinh 1 = e - \frac{1}{e} \quad [2.3504]$

iii) $\int_0^1 \frac{5\sqrt{x} + 7x}{\sqrt[4]{x}} dx = \int_0^1 \left(5x^{\frac{1}{4}} + 7x^{\frac{3}{4}}\right) dx = \left[4x^{\frac{5}{4}} + 4x^{\frac{7}{4}}\right]_0^1 = 8$

iv) $\int_1^e \underset{F'(x)}{x^3}\,\underset{G(x)}{\ln x}\, dx = \left[\frac{1}{4}x^4 \ln x\right]_1^e - \int_1^e \frac{1}{4}x^4 \cdot \frac{1}{x} dx = \frac{1}{4}e^4 - \frac{1}{4}\int_1^e x^3 dx =$

$= \frac{1}{4}e^4 - \frac{1}{16}\left[x^4\right]_1^e = \frac{3}{16}e^4 + \frac{1}{16} \qquad [10.299653]$

v) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{du}{1+u^2} = \left[\arctan u\right]_0^1 = \frac{\pi}{4}$

Substituiere
$u = \cos x$
$du = -\sin x \, dx$

vi) $\int_{3/\pi}^{6/\pi} \frac{1}{x^2} \sin\frac{1}{x} dx = -\int_{\pi/3}^{\pi/6} \sin u \, du = \left[-\cos u\right]_{\pi/6}^{\pi/3} = -\frac{1}{2} + \frac{1}{2}\sqrt{3}$

$= \frac{1}{2}(\sqrt{3} - 1) \quad [0.366025]$

Substituiere
$u = \frac{1}{x}$
$du = -\frac{1}{x^2} dx$

3. Aufgabe:

i) $\int_1^\infty \frac{dx}{x^2}$ $\quad \int_1^R \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_1^R = 1 - \frac{1}{R} \longrightarrow 1$ bei $R \to \infty$.

ii) $\int_0^{\pi/2} \cot x \, dx$ $\quad \int_c^{\pi/2} \cot x \, dx = \int_c^{\pi/2} \frac{\cos x}{\sin x} dx = \left[\ln \sin x\right]_c^{\pi/2} = -\ln \sin c$

kein Grenzwert bei $c \downarrow 0$!

iii) $\int_0^\infty (\cosh x - \sinh x) dx$ $\quad \int_0^R (\cosh x - \sinh x) dx = \int_0^R e^{-x} dx = \left[-e^{-x}\right]_0^R =$

$= 1 - e^{-R} \longrightarrow \underline{\underline{1}}$ bei $R \to \infty$

iv) $\int_0^\infty x^2 e^{-x} dx$ $\quad \int_0^R \underset{G(x)}{x^2} \underset{F'(x)}{e^{-x}} dx = \left[-x^2 e^{-x}\right]_0^R + 2\int_0^R \underset{G(x)}{x} \underset{F'(x)}{e^{-x}} dx =$

$= -\frac{R^2}{e^R} + 2\left[-x e^{-x}\right]_0^R + 2\int_0^R e^{-x} dx = -\frac{R^2}{e^R} - 2\frac{R}{e^R} + 2\left(1 - \frac{1}{e^R}\right) \to \underline{\underline{2}}$ bei $R \to \infty$

v) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ $\quad \int_0^c \frac{dx}{\sqrt{1-x^2}} = \left[\arcsin x\right]_0^c = \arcsin c \longrightarrow \underline{\underline{\frac{\pi}{2}}}$ bei $c \to 1$

4. Aufgabe: $\int_0^1 2^x dx$

i) $\int_0^1 2^x dx = \left[\frac{2^x}{\ln 2}\right]_0^1 = \frac{1}{\ln 2}$ $\quad [1.442695]$

ii) Zerlege $[0,1]$ in n gleichlange Teile $x_l = \frac{l}{n}$ $\quad l = 0, \dots, n$

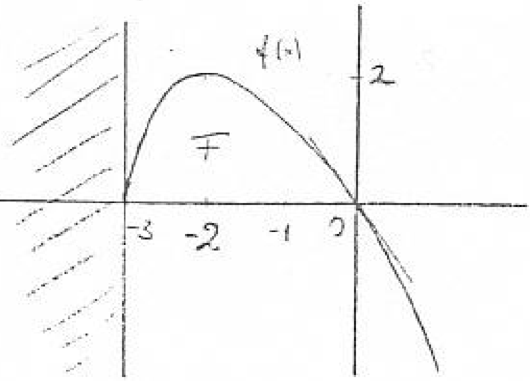
Zerlegungssumme $\sum_{l=0}^{n-1} 2^{x_l}(x_{l+1} - x_l) = \sum_{l=0}^{n-1} \frac{1}{n} 2^{\frac{l}{n}} = \frac{1}{n}\sum_{l=0}^{n-1} \left(2^{\frac{1}{n}}\right)^l =$

$= \frac{1}{n} \frac{\left(2^{\frac{1}{n}}\right)^n - 1}{2^{\frac{1}{n}} - 1} = \frac{1}{\frac{2^{\frac{1}{n}} - 1}{\frac{1}{n}}}$ $\quad \frac{2^x - 1}{x} \longrightarrow ?$ l'Hosp: $\frac{2^x \ln 2}{1} \to \ln 2$

$\longrightarrow \frac{1}{\ln 2}$

5. Aufgabe

$f(x) = -x\sqrt{x+3}$



$$V = \pi \int_{-3}^{0} x^2(x+3)\,dx =$$

$$= \pi \int_{-3}^{0} (x^3 + 3x^2)\,dx =$$

$$= \pi \left[\frac{1}{4}x^4 + x^3\right]_{-3}^{0} = \pi\left(-\frac{81}{4} + \frac{81}{3}\right) = 81\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{27}{4}\pi \quad [21.20575]$$

$$F = -\int_{-3}^{0} x\sqrt{x+3}\,dx = -\int_{0}^{3} (u-3)\sqrt{u}\,du = -\int_{0}^{3}\left(u^{3/2} - 3u^{1/2}\right)du =$$

Substituiere $u = x+3$, $x = u-3$

$$= -\left[\frac{2}{5}u^{5/2} - 2u^{3/2}\right]_{0}^{3} = -\frac{2}{5}\sqrt{3^5} + 2\sqrt{3^3} = -\frac{18}{5}\sqrt{3} + 6\sqrt{3} =$$

$$= \frac{12}{5}\sqrt{3} \quad [4.156922] \qquad \text{Querschnitt} = \frac{24}{5}\sqrt{3}$$

6. Aufgabe:

a) i) $\underbrace{\frac{1}{y}}_{A}dx - \underbrace{\frac{x}{y^2}}_{B}dy$ auf $y > 0$

$\frac{\partial A}{\partial y}(x,y) = -\frac{1}{y^2}$ $\quad \frac{\partial B}{\partial x}(x,y) = -\frac{1}{y^2}$ also totales Diff da Def.ber. ohne Loch

Stammfunktion ist $F(x,y) = \frac{x}{y}$.

ii) $\underbrace{x^2y^2}_{A}dx + \underbrace{x^3y}_{B}dy$ $\qquad \frac{\partial A}{\partial y}(x,y) = 2x^2y$ $\qquad \frac{\partial B}{\partial x}(x,y) = 3x^2y$

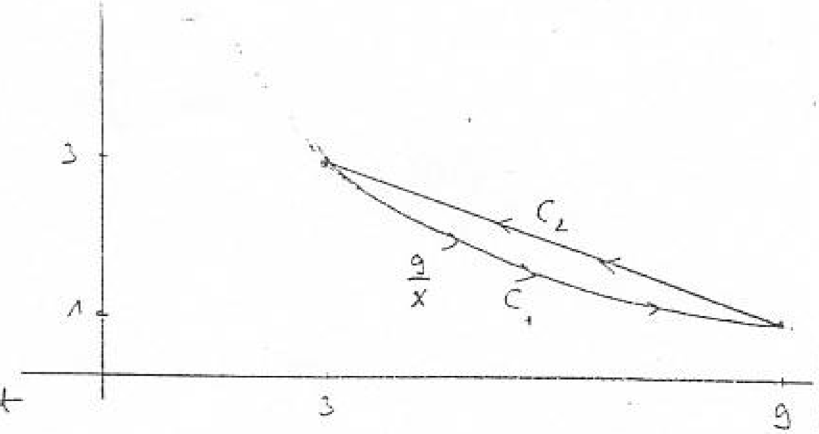
Kein totales Differential

iii) $\underbrace{(x^2y + 2xy^2)}_{A}dx + \underbrace{\left(\frac{1}{3}x^3 + 2x^2y + 4y^3\right)}_{B}dy$

$\frac{\partial A}{\partial y}(x,y) = x^2 + 4xy$ $\qquad \frac{\partial B}{\partial x}(x,y) = x^2 + 4xy$ totales Diff. Def.bereich ohne Loch!

Stammfunktion: $F(x,y) = \frac{1}{3}x^3y + x^2y^2 + y^4$

b) $\int_C (2x+1)\,dx + 2xy\,dy$



C_1: $x(t) = t$

$y(t) = \frac{9}{t}$ $\quad 3 \le t \le 9$

$$\int_{C_1} = \int_3^9 \left((2t+1) - 2 \cdot 9 \frac{9}{t^2} \right) dt$$

$$= \int_3^9 \left(2t + 1 - 162 \frac{1}{t^2} \right) dt = \left[t^2 + t + \frac{162}{t} \right]_3^9 = 81 + 9$$

$$= 81 + 9 + 18 - (9 + 3 + 54) = 42$$

C_2: $x(t) = 9 + (3-9)t = 9 - 6t$ $\quad 0 \le t \le 1$

$y(t) = 1 + (3-1)t = 1 + 2t$

$$\int_{C_2} = \int_0^1 \left((2(9-6t)+1)(-6) + 2(9-6t)(1+2t)2 \right) dt =$$

$$= \int_0^1 \left((18 - 12t + 1)(-6) + 4(9 + 18t - 6t - 12t^2) \right) dt =$$

$$= \int_0^1 \left(72t - 114 + 36 + 48t - 48t^2 \right) dt = \int_0^1 (-48t^2 + 120t - 78)\, dt$$

$$= \left[-16t^3 + 60t^2 - 78t \right]_0^1 = -34$$

$$\int_C = \int_{C_1} + \int_{C_2} = 8$$